OFFLOADING COGNITIVE DEMANDS OF FRACTIONAL TASKS ON WORKING MEMORY THROUGH DRAWINGS

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This study examined the role of student generated drawings to offload cognitive demands of a mathematical problem. We used Unit Transformation Graphs to compare students' thought processes when they had to solve the problem mentally, and when they were allowed to use pen and paper. The results indicated that the possibility to rely on drawings helped the participants to free up working memory resources and complete a cognitively demanding fractional task.

Keywords: Cognition; Learning Theory; Number Concepts and Operations; Problem Solving.

Background

A growing body of research within psychology of mathematics education has been aimed at understanding students' cognitive affordances and constraints in doing mathematics (e.g., Bull & Lee, 2014; De Smedt et al., 2009). In particular, prior literature revealed an important role of working memory (WM) – a psychological construct for human's ability to simultaneously store and process information (Baddeley & Hitch, 1974; Daneman & Carpenter, 1980; Ma et al., 2014). WM is limited and varies widely among individuals. If the complexity of a cognitive task requires to operate with multiple items at the same time, students' WM may be overloaded and they become unable to process the requested amount of information. In this case, it is natural for an individual to seek ways to offload the cognitive demands on their WM. Once information is offloaded into the environment, a student then has more mental space available to progress on task (Kirsh, 2009).

A typical example of cognitive offloading is hand gesturing. For instance, Alibali and DiRusso (1999) investigated the role of gestures on the counting abilities of children. They suggested that finger counting allows children to physically instantiate some aspects of the task and, therefore, offload cognitive demands on their WM. Goldin-Meadow et al. (2001) measured the number of items students can hold in their mind when they are explaining a mathematical task. The results indicated that the participants are able to recall more items when they are allowed to gesture. Similar results supporting the beneficial effect of gesturing on WM were achieved in other studies (Cook et al., 2012; Ping & Goldin-Meadow, 2010; Wagner et al., 2004).

Drawings are considered to be a mediation tool for thinking and for meaning making. Vygotsky and Cole (1978) pointed out that human beings' mental activities are supported and developed by means of signs that are the products of their internalization processes and are called psychological tools. Vygotsky (1981) then suggests a list of examples of psychological tools, including drawings. Therefore, as drawings are composed of systems of signs and, thus, physically represent the original properties of an object of thought, they allow an individual to transmit some pieces of information into the environment and, therefore, reduce cognitive demands, playing the role of an important offloading mechanism.

Empirical research was done to examine the impact of student generated drawings with regards to WM and cognitive offloading on college students' understanding of science text. Lin et al. (2017) explored the levels of WM and the cognitive load that students bear when learning college level science material. In their study, three groups of students were instructed to study in particular ways: one group of students was told to create representations of the material being studied, another had a repeated reading method, and the other group simply had to imagine the relationships and reason through ideas in their heads. Evidence supported the conclusion that the study method of using learner-generated drawings facilitated schema construction to integrate prior knowledge with the new information that allows for easier transfer to storage in long term memory instead of simply with their WM. Learners who exhibited a lower prior knowledge of the course content demonstrated a deeper and higher level of understanding through the use of learner-generated drawings. It also found that students bore less of a cognitive load when they were able to draw out and connect their ideas on paper. By being able to draw out the relationships and make proper connections, students were able to offload the information learned during the study rather than attempting to make all of the connections mentally.

Another study explored the components and effects of learner-generated drawing on subsequent testing. During the learning process, internalizing and externalizing the information can be difficult for students to retain, but once this information is offloaded into the environment it is not as difficult to keep track of (Schmidgall et al., 2019). The process of drawing out representations allows learners to engage in generative learning processes. This study found that the students who utilized the drawing methods for studying outperformed those who simply did a summary of the information to study.

Despite the prior work and findings, drawings have received considerably little attention as being a reliable method for offloading cognitive demands on students' WM when they are engaged in mathematical reasoning. The purpose of the present study is to examine the impact of using student generated drawings to offload the cognitive demands of mathematical tasks. Understanding mechanisms underlying the phenomenon of cognitive offloading will help to improve learning conditions for mathematics learners, which addresses one of the major goals of PME-NA. Specifically, we contribute to the existing body of research by answering the following research question: *How do drawings lighten the load of students' working memory when they are engaged in fractions tasks?*

Theoretical Framework

In answering our research question, we adopt a Piagetian perspective on the construction of mathematical knowledge and a neo-Piagetian approach to WM. Following Piaget, we conceptualize mathematics as a coordination of mental actions (e.g., Beth & Piaget, 1966). In the context of fractional knowledge, these actions are well researched (Boyce & Norton, 2016; Hackenberg & Tilemma, 2009; Steffe & Olive, 2010; Steffe, 1991, 1992). The operations most germane to our study include *unitizing* (U_n; taking a collection of n units as a whole), *iterating* (I_n; copying a unit n times to form a new unit), *partitioning* (P_n; breaking a whole into n identical units); *disembedding* (D_n; taking n equal units out of a whole without losing their connection to the whole), and *distributing* ($T_{m:n}$; inserting a collection of m units of a whole into each of the n units of another whole to create a unit of units of units).

Pascual-Leone (1970) proposed a neo-Piagetian characterization of WM through a mental-attentional operator, which is known as *M*-operator. Its capacity is called *M*-capacity and represents "the number of separate schemes (i.e., separate chunks of information) on which the

subject can operate simultaneously using his mental structures" (p. 302). It has been shown that an average adult's *M*-capacity ranges from 5 to 7, meaning that they can activate 5-7 schemes at once, without offloading cognitive demands onto the environment. In the context of fractional tasks, we hypothesize that WM involves manipulating sequences of actions used to construct and transform units.

Although an average person cannot activate more than 7 schemes during a cognitive task, these schemes differ in complexity and may contain other, "smaller" schemes. In particular, some of the units and unit transformations can be organized within "larger" units coordinating structures (Boyce & Norton, 2016; Hackenberg, 2007; Ulrich, 2016). Consider, for example, the construction of 1/12 as a unit, which has a one-to-twelve relationship to a whole unit. 1/12 may be obtained by partitioning the whole into 12 equal units (see the left part of Figure 1). Vice versa, the whole may be built by the mental action of iterating 1/12 twelve times. In the absence of structures to assimilate multiple levels of units, each unit or unit transformation poses separate demands on a student's WM. However, the whole, the unit fraction, and the mental transformations between them may be chunked into a single cognitive unit, thus reducing cognitive demand of the task from three to one (see the right part of Figure 1).

Figure 1: Units coordinating structures

Our integrated framework explicitly accounts for students' available structures in lightening cognitive load on their WM when solving fractional tasks. However, the increase in the number of units and mental transformations required to operate on may overwhelm one's WM even for students with high WM capacity and advanced level of unit coordination. The utility of drawings in handling the situations of cognitive overload is the primary interest of this study.

Data and Methods

The data used in this paper is part of a larger project aimed to explore behavioral and neurological aspects of mathematical development. Here, we report on video recorded behavioral data and students' written work.

Participants

The participants recruited for this project were pre-service elementary teachers (PSTs) at a large public research university in the southeastern United States, enrolled in one of two sections of Mathematics for Elementary School Teachers, taught by the same instructor. We chose to invite PSTs because they were encouraged to explain and reflect on their reasoning while solving elementary school mathematics tasks. In total, 12 students agreed to participate. In this paper, we report on two of them.

Data Collection and Analysis

Each PST participated in an individual clinical interview (Clement, 2000) with a member of the research team, lasting approximately 75 minutes. The interviews consisted of an assessment of students' available structures (Norton et al., 2015), a WM assessment (backward digit span; Morra, 1994), and a set of fraction tasks (modified from Hackenberg & Tillema, 2009). In this

study, we focus on one of those tasks: *Imagine cutting off 1/4 of 5/6 of a cake. So, how much is that of the whole cake?* The analysis of cognitive demands of this task can be found in (Kerrigan et al., 2020).

The participants were given the fraction task verbally and asked to initially solve it without using any figurative materials, so that we could determine the number of units and unit transformations each student could hold in their WM without relying on drawings. The participants were sometimes asked follow-up questions to clarify their reasoning. None of the PSTs were able to properly solve the task mentally. After they attempted to solve the fraction task in their heads, the participants were given the opportunity to use a pen and paper. All interviews were video and audio recorded, and all drawings were captured using LiveScribe pens.

We qualitatively analyzed the behavioral video data, transcripts of students' verbal reasoning, and data collected via LiveScribe pens. We built unit transformation graphs (UTGs; Norton et al., in press) to illustrate the sequences of actions the students' used in attempting to solve the task mentally, and being able to rely on drawings. The comparisons of these graphs helped us to draw inferences about the impact of student generated drawings on cognitive offloading for fraction tasks.

Results

We report on two participants: PST A (with WM capacity of 6, operating at unit coordination stage 3) and PST B (with WM capacity of 7, operating at units coordination stage 2). We decided to choose these two students because both of them were assessed with high WM capacities, made the most progress on the task in the initial phase, and could come up with the correct answer when they were allowed to use drawings. Here, we present our analysis of their reasoning before and after drawing.

PST A Before Drawing

Researcher: This is the next task. Imagine cutting off one fourth of five sixths of a cake. So, how much is that of the cake. First just try to do it in your head then you can draw.

PST A: Ok, so [motions with hands] three fourths of the five sixths.

Researcher: One fourth of five sixths.

PST A: One fourth. One fourth of the five sixths. Ok, so I see six bars, but only five. So, one fourth of one… One fourth of the five sixths… Ok…. So, that would be… [pauses for twenty-six seconds.] Umm… Five… Twenty fourths? But you're not using all six… It's really hard to do in your head. I'm not sure. I'm like really messing up your experiment, I'm sorry.

Researcher: No, this is… I mean this is… We have to have ones where you're…

PST A: Just complete dud. Um, so you have five sixths [Student motions with hands]. And one fourth of that… The fifths don't go in evenly with the fourths. So that's what is confusing to me. Is there some easy way to multiply these? Am I just, am I overlooking something?

Researcher: No.

PST A: Um…

Researcher: Do you want to draw it?

PST A: Yeah.

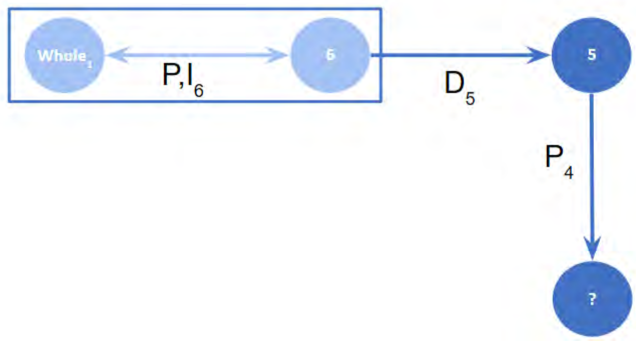

Figure 2: UTG for PST A before drawing.

As evident in UTG (Figure 2), we inferred that PST A used a two-level unit coordinating structure. This means that she could partition the whole into 6 equal parts, while being simultaneously aware of the whole, a piece of the whole, and a 1:6 relationship between them (see the rectangle in Figure 2). Therefore, mental actions of partitioning and iterating, as well as the two quantities they connect, allowed the student to chunk these three units, reducing the cognitive demand of the task by two. She could further disembed 5/6 and attempted to operate on it through partitioning into four parts. However, the latter action led to considerable cognitive overload, which might impede the student's progress on the task.

PST A After Drawing

PST A: [Picks up pen and begins to draw.] So, you have six things… Or six… But you're only using five of them. So, you need one fourth of this.
[Student thinks and draws for fourteen seconds.] Split that into fours.
Researcher: Mhm.
Student: Ok so that means it's one-fourth of that, so… One two three four five. Of the whole thing? No, just five sixths. So, five…
Researcher: Out of the- uh- original cake.
PST A: So, six sixths?
Researcher: Yes.
PST A: So, five out of, five twenty-fourths?
Researcher: Yeah.
PST A: Five twenty-fourths.

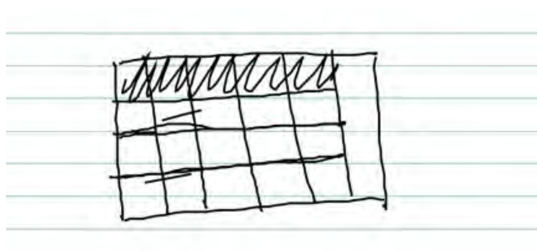

Figure 3: LiveScribe Pen drawing of PST A after drawing.

Being able to draw the problem out allowed PST A to complete the task. The student first drew a whole and split it into six pieces. She then cut each of the five pieces into four parts. After

that, she shaded in a fourth from each fifth, as shown in Figure 3. Finally, the participant counted the total number of fours in each sixth to find the total number of pieces to base her shaded portion out of.

PST B Before Drawing

PST B: You're cutting off one fourth of five sixths of a cake?

Researcher: Yes.

PST B: [uses hands to show number of pieces on the desk and begins talking to herself] So, you'd have, so you'd have six pieces…and out of those five…you want to cut off one fourth of that. Um…I guess you would…I mean I guess you could split those five pieces into four and get one of those, but I'm trying to think like numbers-wise what that would...I. well… [pauses for seven seconds] I guess of those five pieces you could…Split them into…Like you could get a…Split them into twenty pieces because five times four is twenty and then, um, you would take one fourth of that… I guess it would be five pieces. Yeah, it would be five pieces of that twenty to find the one fourth of the five sixth. Is that, do I need to explain it more?

Researcher: Okay, uh let's…

PST B: Which would be, do you want me to draw it? [reaches towards paper]

Researcher: Well tell me the final answer and then we can draw it.

PST B: Um, oh gosh it would be… [pauses for four seconds] Splitting twenty, it would be five…Well it would be five twentieths, which would equal one fourth, so like five of those, but then I don't know how to figure that out into sixths. I think that's my…

Researcher: Yeah that's cool, I like the way you're reasoning. Let's draw it, and I think you will figure it out.

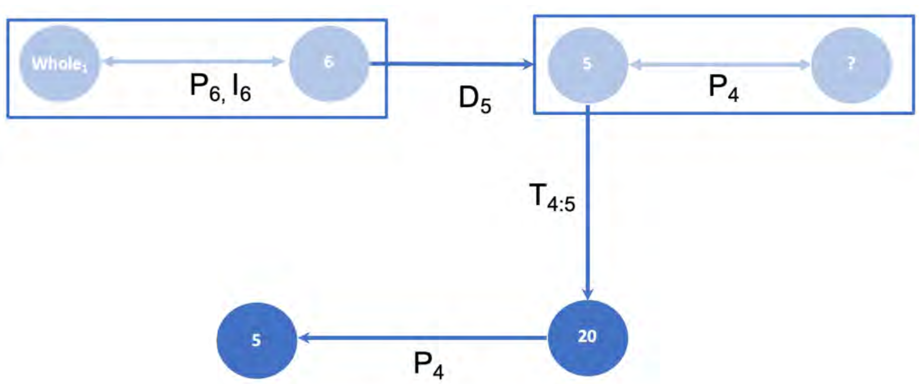

Figure 4: UTG for PST B before drawing.

Similar to PST A, PST B began with the use of a two-level structure to conceptualize 1/6, then disembedded 5 copies of it ("so you'd have six pieces… and out of those five…"), and attempted to split them into four pieces (Figure 4). However, when constructing 1/4 of 5/6, the student lost track of the sixth part making up the whole. She, thus, ended up distributing four parts into each of the 5 pieces, producing 20 parts in total. Taking a quarter out of 20, with no relation to the whole, resulted in her final answer of 5/20, or 1/4.

PST B After Drawing

PST B: Okay, so there's…This and then…Five, and we're trying to find… We have five sixths and we're trying to find one-fourth, and you… [Student works for fifteen seconds.] Okay, wait can I… can I cross this out?

Researcher: Sure. [Student crosses out picture.]

PST B: Okay this is five sixths, we want five of those and then you would, I guess divide…

[Student writes on paper for twenty-four seconds.] So then yeah you would have twenty little pieces and then…One fourth of that…Would just be, one two three four five. [Student marks pieces.] So… so I guess in terms of sixths it would be like…Well, oh wait I guess, wait…In total it would be, twenty-five twenty-fifths…Okay so in terms of six that would be… [Student works for six seconds.] Five…Wait, yeah it would be five twenty-fifths would be one fifth. I don't think that's right.

Researcher: Why don't you think it's right?

PST B: I just think, I just don't think one fourth of… I don't know it just doesn't…

Researcher: How did you get the twenty-five?

PST B: Um, I shaded in… Well I shaded in five of these because that's one fourth of the five sixths, because there are… [Student counts on drawing.] Because that's one fourth of that and then, when you turn the five sixths into terms of…Um…Twentieths, well, then I just… [Student reexamines paper.] …There are actually twenty-four…There's… So, it's out of twenty-four then, I guess, so then it would be, five twenty-fourths.

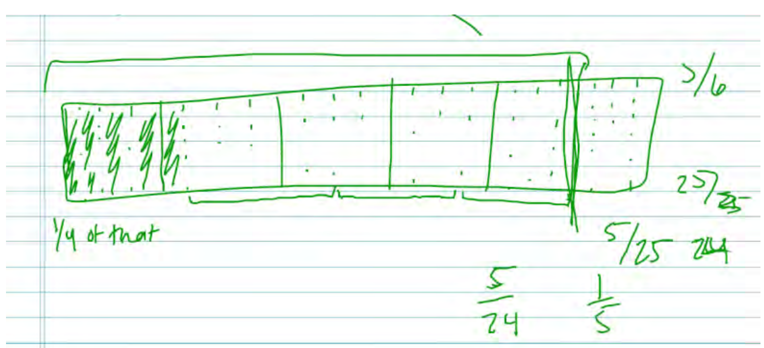

Figure 5: LiveScribe Pen drawing of PST B.

Consistent with her original strategy, the participant partitioned the whole into six pieces and marked off five of them. From there, she went through her previous approach of fourthing 5/6 by dividing each fifth piece into four smaller pieces to make the total of twenty within the fives.

Taking 1/4 of that, the student shaded in five pieces of that twenty, aligning with her previous final answer (Figure 5). However, this time, she was able to refer back to comparing that new piece to the whole cake by dividing the remaining 1/6 into four parts as well and counting the total number of pieces. The ability to draw the problem out and to refer to the depiction allowed PST B to maintain her perspective on the whole without losing the extra 1/6, and eventually come up with the correct answer of 5/24.

Discussion

The partial solutions, presented by our participants before drawing, exhausted their WM capacities. Both of the students employed two-level units coordinating structures, which allowed them to alleviate the cognitive demands on their WM. However, the experienced cognitive demand of the task was too high, leaving no cognitive resources required to complete the task. PST A had potential to make greater progress on the task because she was assessed as having WM capacity of 6 and operating at unit coordination stage 3. Nevertheless, it is known that students do not necessarily construct all possible mental structures by virtue of operating at stage 3 (Izsák, 2008). Similarly, it appears that people do not always use all their available WM resources (Eysenck & Cavo, 1992; Norton et al., in press).

After being able to represent their thought processes pictorially, both students could continue with their original method of solving the problem. Although PST A and PST B produced different drawings, their final thought processes converged (see Figure 6). They both partitioned the whole into 6 pieces, disembedded 5 of them, which constituted a new whole. After that, they distributed 4 pieces into each of the 5 pieces, producing 20 parts, which were further partitioned into 4 parts, arriving at 5 smaller pieces. In order to conceptualize how much of the whole cake these 5 pieces were, the participants had to repeat the process for the remaining 1/6 (indicated by dashed lines, ovals, and cross-hashed circles in Figure 6). Because the first part of the solution was already drawn out, the students could focus on the second part without losing any of the previously obtained information. Thus, the possibility of relying on the picture allowed the participants to save mental resources and eventually come up with the correct answer, as predicted by the prior literature (Kirsch, 2009; Schmidgall et al., 2019).

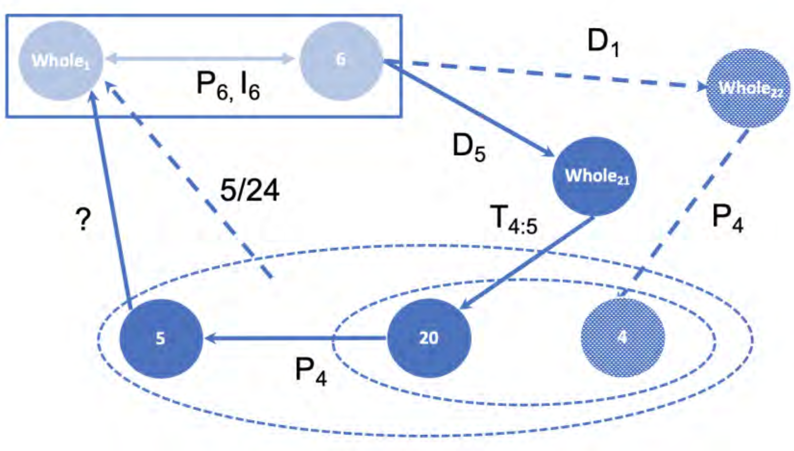

Figure 6: UTG after drawing for both PST A and PST B.

Notably, in absence of other offloading tools, the participants actively produced hand gestures when reasoning through the task. Gesturing was used as a natural method to physically represent some properties of the object of thought and make the task less demanding (e.g., Ping & Goldin-Meadow, 2010; Wagner et al., 2004). However, the use of hand gestures did not help our participants to overcome the experienced cognitive overload. Future research should be conducted to compare the effects of gesturing and drawing on managing the cognitive demands of mathematical tasks.

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